

The Basic Physics of Gyroscopic Hand Exercisers

Nicholas A. Masluk
nick@randombytes.net

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Using elementary mechanics, the basic principles involved in the operation of gyroscopic exercisers can be explained. The exerciser consists of a rotor housed in a shell, where the axle of the rotor sits in a groove that goes around the circumference of the shell. The width of the groove is a bit larger than the axle diameter, and it allows the axle to rotate transversely to its axis. It is important to note is that there is friction between the groove and axis; this is how energy is transferred to the rotor, as it allows us to apply a torque to the axle along its axis. If one were to lubricate the groove, the exerciser would become difficult if not impossible to operate.

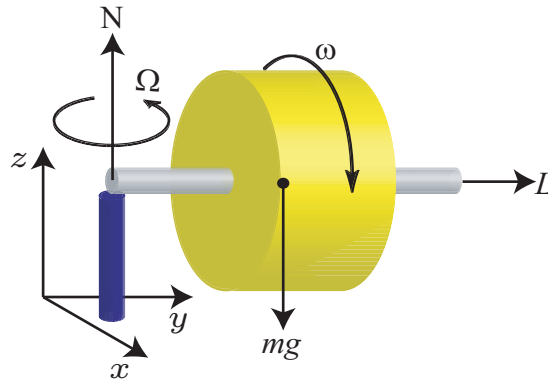


Figure 1: **Top on a stand**

Let's first consider a symmetric top on a stand, as shown in Fig 1, with mass m and an angular velocity ω about a moment of inertia I . Here gravity produces a torque $\vec{\tau} = \vec{r} \times m\vec{g}$ perpendicular to the angular momentum \vec{L} of the top. This perpendicular torque will only act to change the direction of \vec{L} , as it is always perpendicular to \vec{L} . This results in a precession about the stand. The change in \vec{L} with time is given by

$$\frac{d\vec{L}}{dt} = \vec{\tau} \quad (1)$$

or

$$dt = \frac{d\vec{L}}{\vec{\tau}} \quad (2)$$

and the change in axis angle in the xy plane from precession is given by

$$d\phi = \frac{|d\vec{L}|}{|\vec{L}|} \quad (3)$$

So, the precession angular frequency is given by

$$\Omega = \frac{d\phi}{dt} = \frac{|d\vec{L}|/|\vec{L}|}{d\vec{L}/\vec{\tau}} = \frac{\tau}{L} = \frac{\tau}{I\omega} \quad (4)$$

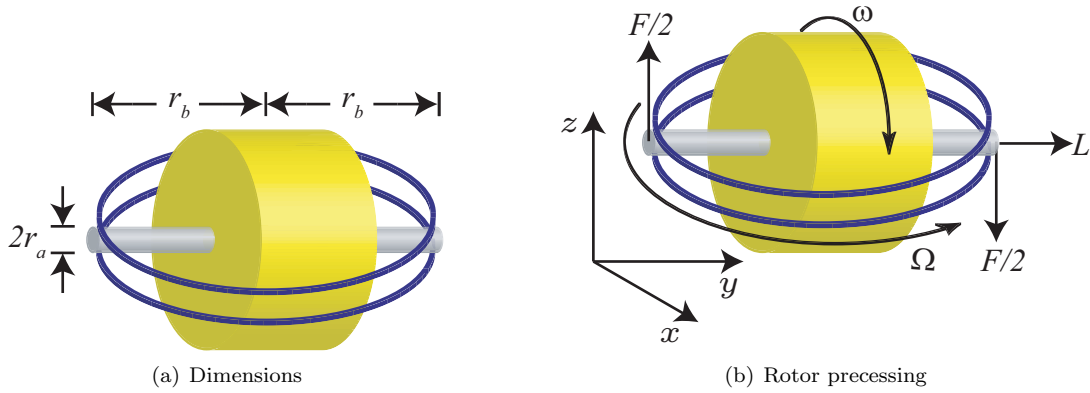


Figure 2: **Exerciser rotor and shell groove**

In the exerciser, the force which generates precession comes from the user. Gravity does not play a role, as the rotor is supported on both ends of the axle. Because force must be applied perpendicular to the axle to provide a precession generating torque, the force the user must apply rotates with the axle at the precessional rate. The rotor and groove from the shell are shown in Fig 2(a). We let I be the moment of inertia about the axle, ω be the angular velocity around the axle, r_a be the axle radius, r_b be the length from the center of the axle to the end, Ω be the precession angular velocity, and F be the total force applied perpendicular to the axle at the radius of the shell's groove, producing a torque $\tau = Fr_b$.

As the user forces the rotor to precess, its axle rubs against the side of the groove in the shell. The contact point between the rotor axle and groove is such that as the rotor is forced to precess, the rotor rubs against the groove in the direction that the torque applied along the axle from friction increases the angular velocity ω of the rotor. For example, take Fig 2(b), where the rotor is spinning such that \vec{L} is in the $+\hat{y}$ direction. A force is applied to the ends of the axle via the groove in the shell, such that there is a force in the $+\hat{x}$ direction on the left axle, and $-\hat{x}$ direction on the right. This means the bottom of the groove is in contact with the left axle and the top of the groove contacts the right axle. The total force applied perpendicular to the axle produces a torque in the $-\hat{x}$ direction, so the rotor precesses in the $+\hat{z}$ direction. We can now see that friction between the axle and groove will reinforce the rotor's angular velocity ω in the $+\hat{y}$ direction, so long as the following condition is met: $r_b\Omega > r_a\omega$. This inequality simply means the precession is fast enough that it will add energy to the rotor, and is found by comparing the linear velocity at the point of contact from precession and the spinning rotor. If one were to reverse the direction force is applied, the axle

will swap the sides of the groove it rubs against, so it doesn't matter which way the rotor precesses (this can be easily demonstrated).

Using equation 4, we can determine what force the user must apply perpendicular to the axle to sustain some precessional rate. Neglecting any losses due to friction or drag, for sustained operation we have

$$\Omega = \frac{Fr_b}{I\omega} \quad (5)$$

We require $r_b\Omega > r_a\omega$ for forced precession to increase ω , the rotor's angular velocity. In this regime

$$\frac{Fr_b}{I\omega} > \frac{r_a\omega}{r_b} \quad (6)$$

$$F > \frac{r_a I \omega^2}{r_b^2} \quad (7)$$

For sustained operation, the inequality can simply be replaced with an equal sign. This expression illustrates the relationship between the various parameters in the system, and how they affect the amount of force the user must apply.